Lesson 4: 4.4 – The Quadratic Formula Part 1

All quadratic equations, can be solved using a formula developed by solving the standard quadratic equation \( ax^2 + bx + c = 0 \) by completing the square. If we followed our steps in solving by completing the square, we would generate the quadratic formula...

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

E.g. Solve the following quadratic by completing the square using the second method from lesson 3. Leave the answers in exact form.

\( 3x^2 + 8x - 3 = 0 \)  \( (a=3 \quad b=8 \quad c=-3) \)

\[
\begin{align*}
3x^2 + 8x - 3 &= 0 \\
\frac{3x^2}{3} + \frac{8x}{3} - \frac{3}{3} &= 0 \\
x^2 + \frac{8}{3}x - 1 &= 0 \\
x^2 + \frac{8}{3}x &= 1 \\
x^2 + \frac{8}{3}x + \frac{64}{36} &= 1 + \frac{64}{36} \\
(x + \frac{8}{6})^2 &= \frac{100}{36} \\
x + \frac{8}{6} &= \frac{\sqrt{100}}{6} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{align*}
\]
Example 1: Learning How to Use the Quadratic Formula

a) What are the values of $a$, $b$, and $c$ in the equation $x^2 + 3x + 2 = 0$?

\[ a=1 \quad b=3 \quad c=2 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(2)}}{2(1)} \]

\[ = \frac{-3 \pm \sqrt{9 - 8}}{2} \]

\[ = \frac{-3 \pm \sqrt{1}}{2} \]

\[ = \frac{-3 \pm 1}{2} \]

\[ x = \frac{-3 + 1}{2}, \frac{-3 - 1}{2} \]

\[ = -1, -2 \]

b) Substitute the values from question part a into the quadratic formula to solve the equation $x^2 + 3x + 2 = 0$.

\[ x = \frac{-3 \pm \sqrt{1}}{2} \]

\[ = \frac{-3 + 1}{2}, \frac{-3 - 1}{2} \]

\[ = -1, -2 \]

b) Check that the solutions are correct; check using factoring or substitution.

**Factoring (not always possible)**

\[ x^2 + 3x + 2 = 0 \]

\[ (x+2)(x+1) = 0 \]

\[ x+2 = 0, \quad x+1 = 0 \]

\[ x = -2, \quad x = -1 \]

**Substitution (always possible)**

\[ f(-1) = (-1)^2 + 3(-1) + 2 \]

\[ = 1 - 3 + 2 = 0 \]

\[ f(-2) = (-2)^2 + 3(-2) + 2 \]

\[ = 4 - 6 + 2 = 0 \]
Example 2: Learning How to Use the Quadratic Formula

Solve each of the following quadratic equations by using the quadratic formula. Verify your answers using the factoring or substitution.

a) \( x^2 + 4x + 3 = 0 \)
\[ a=1 \quad b=4 \quad c=3 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-4 \pm \sqrt{16 - 4(1)(3)}}{2} \]
\[ = \frac{-4 \pm \sqrt{16 - 12}}{2} \]
\[ = \frac{-4 \pm \sqrt{4}}{2} \]
\[ = \frac{-4 \pm 2}{2} \]
\[ = -1, -3 \]

b) \( x^2 + x - 12 = 0 \)

\[ x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-12)}}{2(1)} \]
\[ = \frac{-1 \pm \sqrt{49}}{2} \]
\[ = \frac{-1 \pm 7}{2} \]
\[ = 3, -4 \]

C) \( x^2 + 4x - 5 = 0 \)

\[ x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)} \]
\[ = \frac{-4 \pm \sqrt{16 + 20}}{2} \]
\[ = \frac{-4 \pm \sqrt{36}}{2} \]
\[ = \frac{-4 \pm 6}{2} \]
\[ = 1, -5 \]
Example 3: Rational Roots

a) Solve $3x^2 + 5x - 2 = 0$. Provide the answer in simplified form and as a decimal approximation.

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$= \frac{-5 \pm \sqrt{49}}{6}$$

$$= \frac{-5 \pm 7}{6}$$

$$x = \frac{2}{6}, \frac{-12}{6}$$

exact

$$x = \frac{1}{3}, -2$$

or

$$x = 0.3, -2$$

decimal approximation.

b) Use the solutions to help you graph the corresponding quadratic function.
Example 4: Irrational Roots

a) Solve and check \( x^2 - 2x - 1 = 0 \). Provide the answer in simplified form and as a decimal approximation.

\[
x = \frac{-(2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}
\]

\[
x = \frac{2 \pm \sqrt{8}}{2}
\]

\[
x = \frac{2 \pm 2\sqrt{2}}{2}
\]

\[
x = 1 \pm \sqrt{2}
\]

or

\[
x = 1 + \sqrt{2}, 1 - \sqrt{2}
\]

\[
x = -0.414, 2.41
\]

\[
\text{simplified/exact form.}
\]

\[
\text{decimal approximation.}
\]

b) Use the solutions to help you graph the corresponding quadratic function.
Example 5: Applications

You want to frame a painting that is painted on a canvas measuring 50 cm by 60 cm. Before framing, she places the painting on a rectangular mat so that a uniform (same width) strip of the mat shows on all sides of the painting. The area of the mat is twice the area of the painting. How wide is the strip of exposed mat showing on all sides of the painting, to the nearest tenth of a centimetre?

\[
\begin{align*}
\text{Picture} \quad A &= 3000 \text{ cm}^2 \\
\text{Mat} \quad A &= 6000 \text{ cm}^2
\end{align*}
\]

\[
(60+2x)(50+2x) = 6000
\]

\[
3000 + 120x + 100x + 4x^2 = 6000
\]

\[
4x^2 + 220x + 300 - 6000 = 0
\]

\[
4x^2 + 220x - 3000 = 0
\]

\[
x = \frac{-220 \pm \sqrt{(220)^2 - 4(4)(-3000)}}{2(4)}
\]

\[
x = \frac{-220 \pm \sqrt{96400}}{8}
\]

\[
x = \frac{-220 \pm \sqrt{96400}}{8}
\]

\[
x = \frac{-220 \pm 980}{8}
\]

\[
x = \frac{-220 + 980}{8} \quad \text{or} \quad \frac{-220 - 980}{8}
\]

\[
x = \frac{760}{8} \quad \text{or} \quad \frac{-1200}{8}
\]

\[
x = 95 \quad \text{or} \quad -150
\]

\[
\therefore \text{the width of the exposed mat is about } 11.3 \text{ cm.}
\]

\[
\text{**Notice if we substitute this value to create the total mat area of 6000, we get } 5996.76 \text{ instead. Can you explain why?}
\]

end of Lesson 4 - 28